Complex Dynamic Networks: Architectures, Games, Components, Probability

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Taxonomy of Networked Systems

Infrastructure / Communication Networks

Internet / WWW
MANET
Sensor Nets
Robotic Nets
Hybrid Nets: Comm, Sensor, Robotic and Human Nets

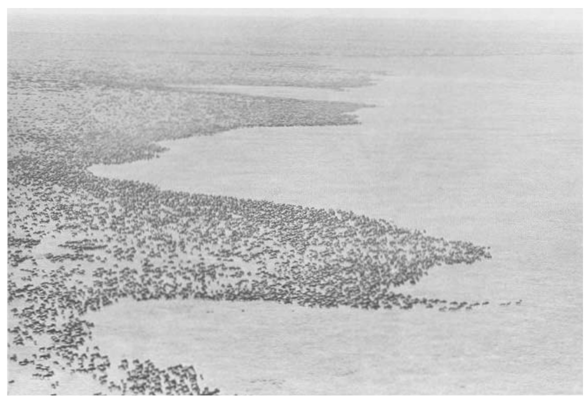
Social / Economic Networtks

Social Interactions
Collaboration
Social Filtering
Economic Alliances
Web-based social systems

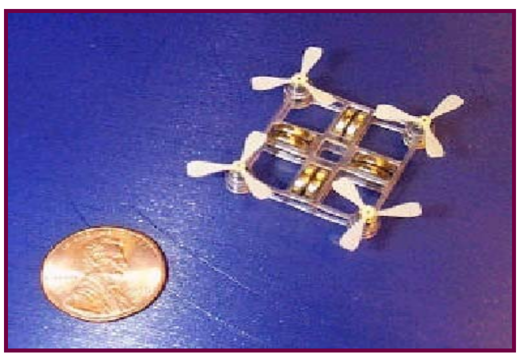
Biological Networks

Community
Epidemic
Cellular and Sub-cellular
Neural
Insects
Animal Flocks

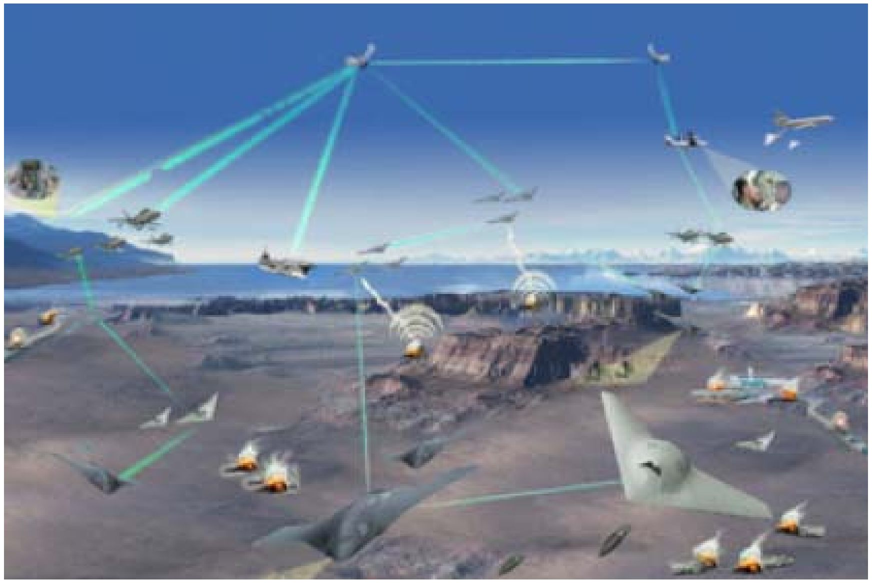
Biological Swarms

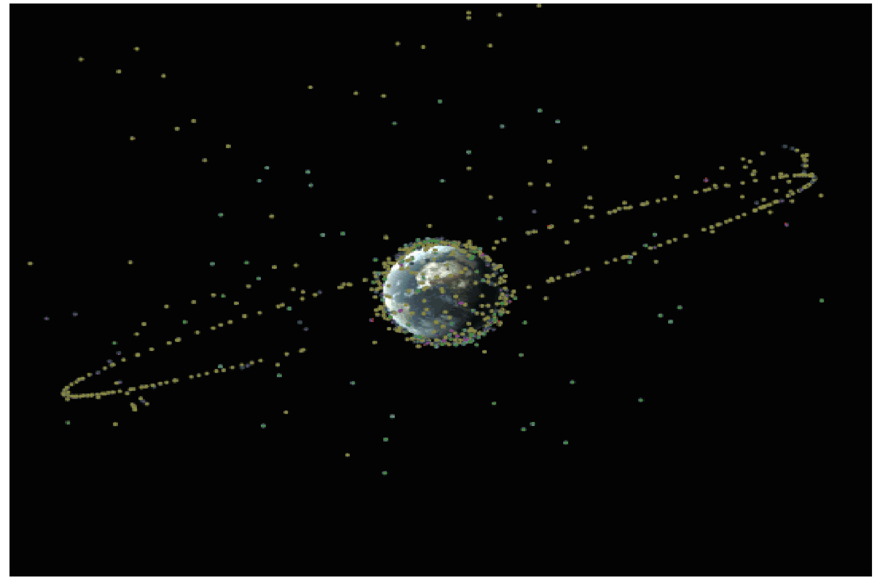


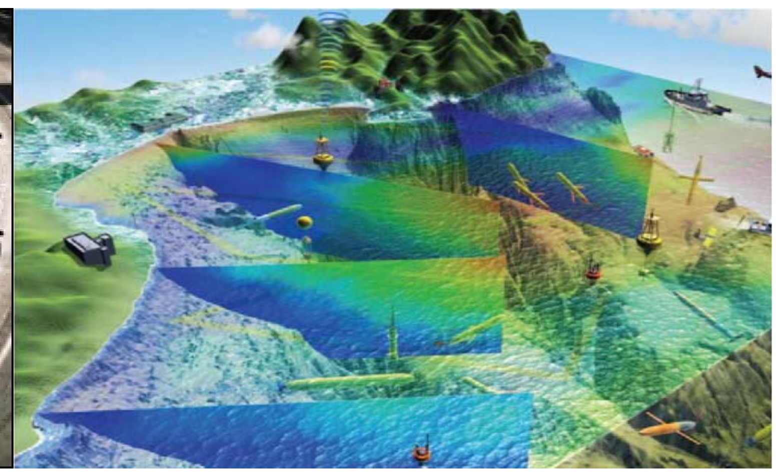
Collaborative Robotic Swarms



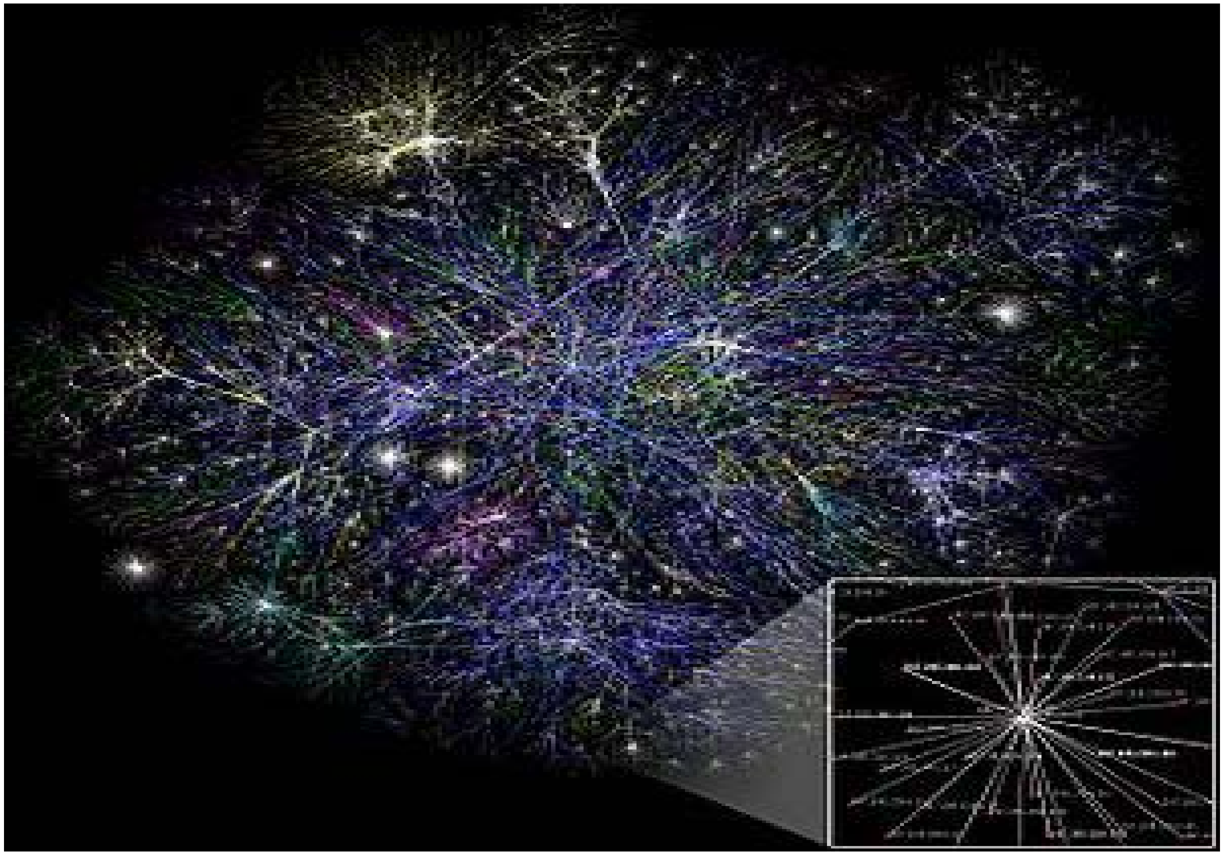
Autonomous Swarms – Networked Control



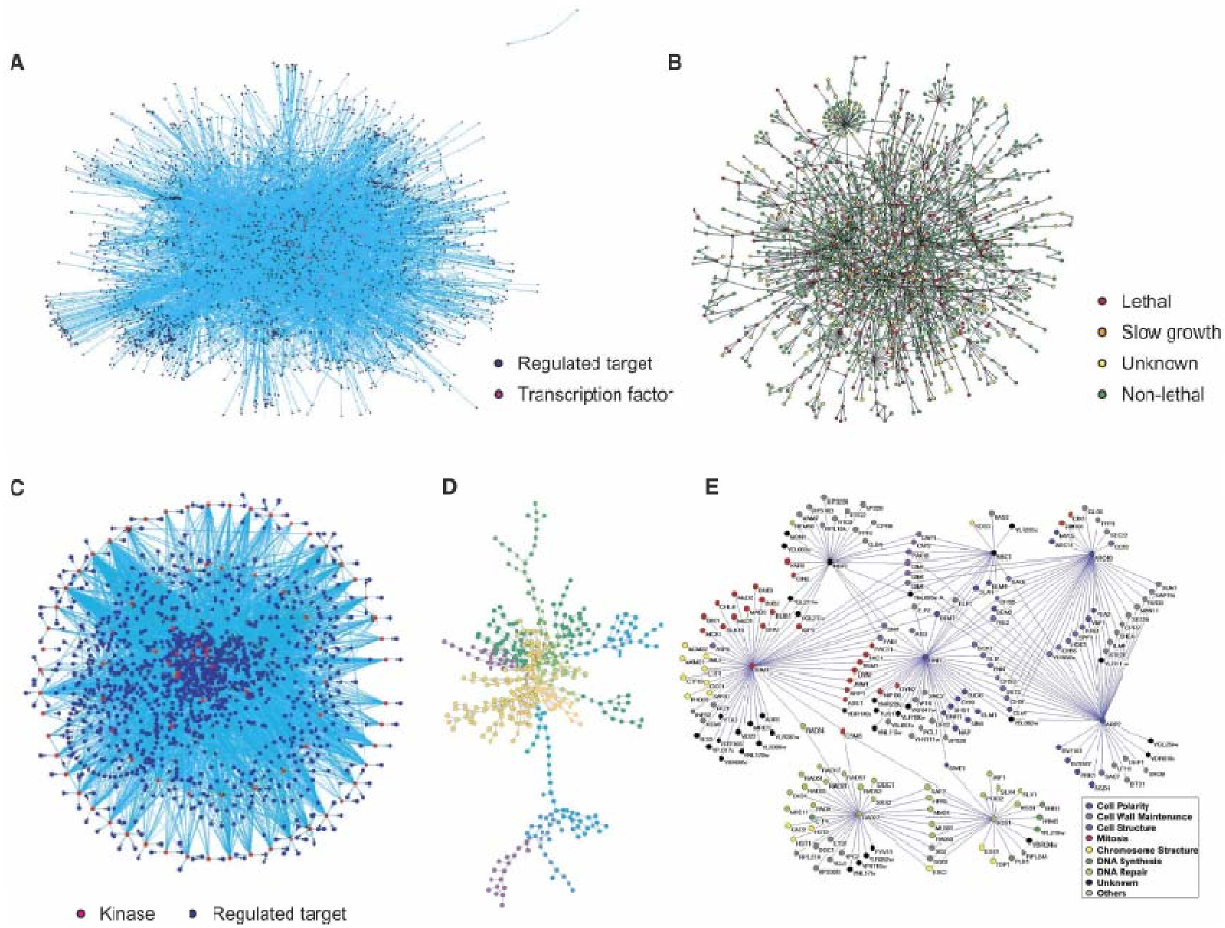




The Internet



Biological Networks

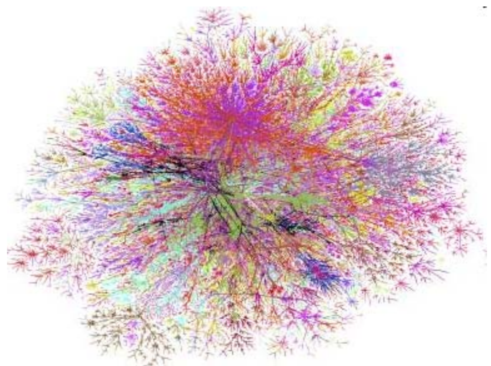


Examples of biological networks: [A] Yeast transcription factor-binding network; [B] Yeast protein -protein interaction network; [C] Yeast phosphorylation network ; [D] *E. Coli* metabolic network ; [E] Yeast genetic network ; Nodes colored according to their YPD cellular roles [Zhu et al, 2007]

The Institute for Systems Research

Networks and Networked Systems

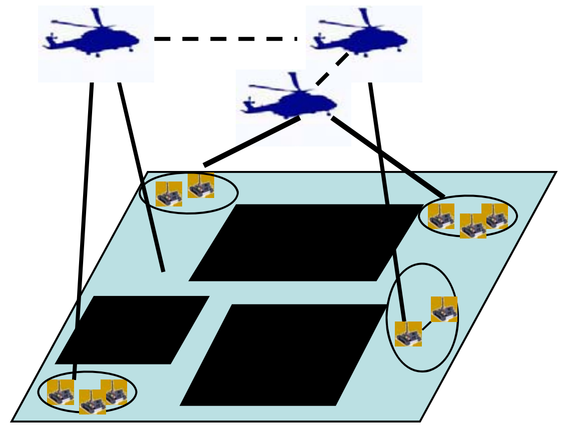
UNIVERSITY OF MARYLAND 1856



Internet backbone

(Lumeta Corp.)

Physical

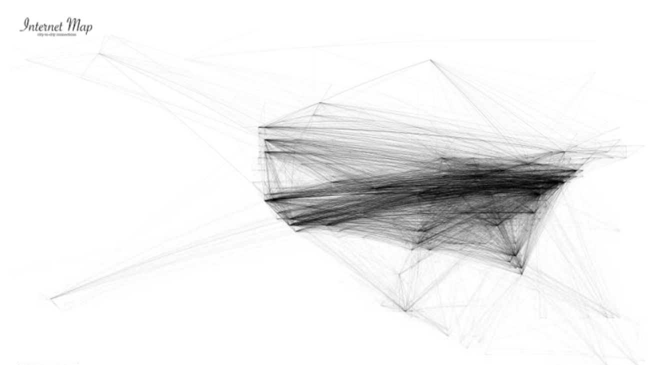


Vehicle, robot networks

Trust

(J Golbeck - Science, 2008)

Logical



Internet: North American cities

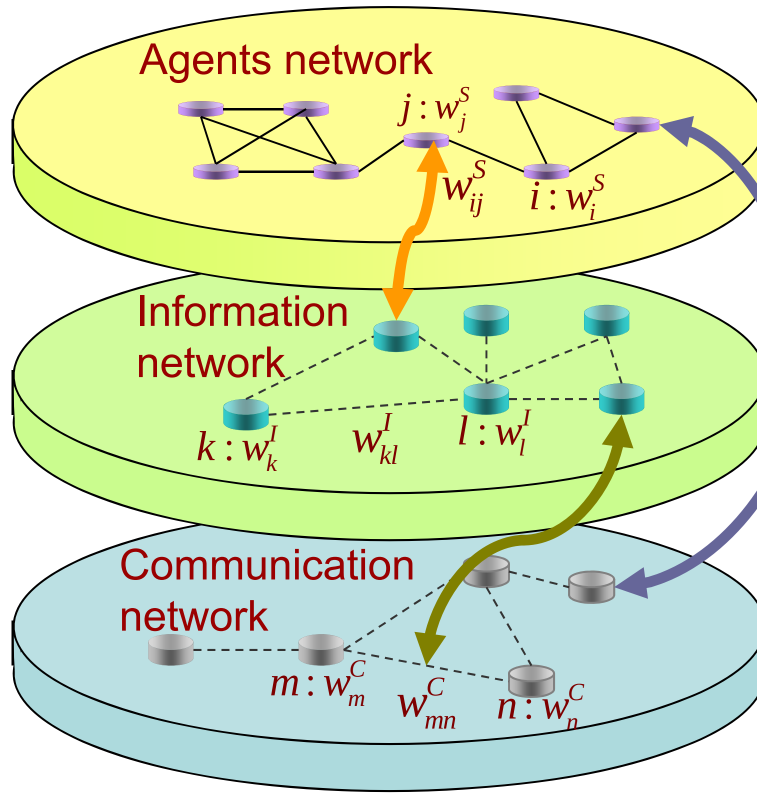
(Chris Harrison)

Outline

- Multiple interacting dynamic hypergraphs – four challenges
- Networks and Collaboration -- Constrained Coalitional Games
- Trust and Networks
- Component-based network synthesis
- Topology and performance
- New probability models (non Kolmogorov)
- Biological networks and cancer dynamics
- Conclusions and Future Directions

Multiple Interacting Dynamic Hypergraphs

- Multiple Interacting Graphs
  - *Nodes*: agents, individuals, groups, organizations
  - Directed graphs
  - *Links*: ties, relationships
  - Weights on links : value (strength, significance) of tie
  - Weights on nodes : importance of node (agent)
- **Value directed graphs with weighted nodes**
- **Real-life problems: Dynamic, time varying graphs, relations, weights, policies**



Networked System architecture & operation

Network Complexity: Four Fundamental Challenges

- **Multiple interacting dynamic hypergraphs involved**
  - ***Collaboration hypergraph***: who collaborates with whom / when
  - ***Communication hypergraph***: who communicates with whom / when
- **Effects of connectivity topologies**:
  Find graph topologies with favorable tradeoff between performance (**benefit**) *vs* **cost** of collaborative behaviors
  - **Small word graphs** achieve such **tradeoff**
- Components, Interfaces, **Compositional Synthesis**
  - Network protocols – **component based networing**
  - Compositional Universal Security
- Need for **different probability models** – the classical Kolmogorov model is **not correct**
  - Probability models over logics and timed structures
  - **Logic of projections in Hilbert spaces** – not the Boolean of subsets

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What is a Network …?

- **In several fields or contexts:**
  - **social**
  - **economic**
  - **communication**
  - **sensor**
  - **biological**
  - **physics and materials**

A Network is …

- A collection of nodes, agents, …

  that **collaborate** to accomplish actions, gains, …

  that cannot be accomplished with out such collaboration

- Most significant concept for **dynamic autonomic networks**

The Fundamental Trade-off

- The nodes **gain** from collaborating
- But collaboration has **costs** (e.g. **communications)**
- **Trade-off: gain from collaboration *vs* cost of collaboration**

  Vector metrics involved typically

  Constrained Coalitional Games

- **Example 1**: **Network Formation** -- Effects on Topology
- **Example 2**: **Collaborative robotics, communications**
- **Example 3**: **Web-based social networks and services**
- **Example 4**: **Groups of cancer tumor or virus cells**

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Example: Autonomic Networks

- Autonomic: self-organized, distributed, unattended
  - Sensor networks
  - Mobile ad hoc networks
  - Ubiquitous computing
- Autonomic networks depend on **collaboration** between their nodes for all their functions
  - The nodes gain from collaboration: e.g. multihop routing
  - Collaboration introduces cost : e.g. energy consumption for packet forwarding

Example: Social Webs

- In August 2007, there were totally 330,000,000 unique visits to social web sites. (Source: Nielsen Online)
  - 9 sites with over 10,000,000 unique visits
  - MySpace, Facebook, Windows Live Spaces, Flickr, Classmates Online, Orkut, Yahoo! Groups, MSN Groups

- Main types of social networking services
  - directories of some categories: e.g. former classmates
  - means to connect with friends: usually with self-description pages
  - **recommender systems** linked to **trust/reputation**

Gain

- Users gain by joining a coalition
  - **Wireless networks**
    - The benefit of nodes in wireless networks can be the rate of data flow they receive, which is a function of the received power

$$B_{ij} = f(P_j l(d_{ij}))$$

P_j is the power to generate the transmission and $l(d_{ij}) < 1$ is the loss factor

e.g: $B_{ij} = \log(1 + (P_j l(d_{ij}) / N_0))$

  - **Social connection** model (Jackson & Wolinsky 1996)

$$B_{ij} = \sum_{j \in g} V \delta^{r_{ij} - 1} \quad \text{or} \quad w_i(G)$$

    - r_{ij} is # of hops in the shortest path between i and j
    - $0 \leq \delta \leq 1$ *is the connection gain depreciation rate*

Cost

- Activating links is costly. $c_i(G) = \sum_{j \in N_i^t} C_{ij}$
  - **Wireless networks**
    - Energy consumption for sending data: $C_{ij} = RSd_{ij}^{\alpha}$

      RS depends on transmitter/receiver antenna gains and system loss not related to propagation

      α : path loss exponent
    - Data loss during transmission

      v_i is the environment noise and I_{ij} is the interference

      $$C_{ij} = h(v_i, I_{ij}) > 0$$
  - **Social connection model**
    - The more a node is **trusted**, the lower the cost to establish link

      e.g.suppose that the trust i has on j is s_{ij} (between 0 and 1), we can define the cost as the inverse of the trust values

      $$C_{ij} = 1 / s_{ij}$$

Coalitional Games

- **Payoff** of node $\boldsymbol{i}$ from the network G is defined as

$$v_i(G) = \text{gain} - \text{cost} = w_i(G) - c_i(G)$$

- **Iterated process**
  - Node pair $\boldsymbol{ij}$ is selected with probability $\boldsymbol{p_{ij}}$
  - If link $\boldsymbol{ij}$ is already in the network, the decision is whether to sever it, and otherwise the decision is whether to activate the link
  - The nodes act **myopically**, activating the link if it makes each at least as well off and one strictly better off, and deleting the link if it makes either player better off
  - **End**: if after some time, no additional links are formed or severed
  - **With random mutations** , the game converges to a unique Pareto equilibrium (underlying Markov chain states)

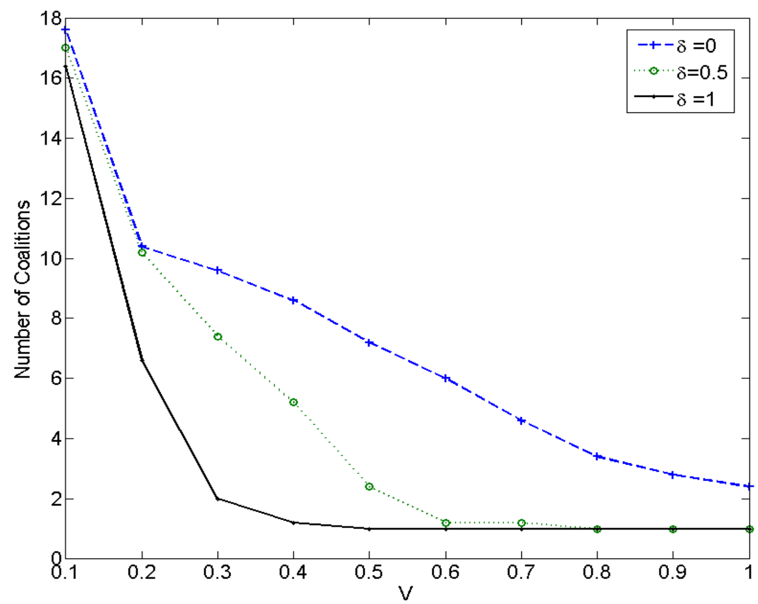
Stochastic Stability

- Dynamic process is now a finite state, aperiodic, irreducible Markov chain (graph process)-- steady-state distribution, $\Pi(g, \varepsilon)$.
- A network *g* is **stochastically stable** if $\Pi(g, \varepsilon)$ is bounded below as the error rate, ε, tends to zero;

  $\Pi(g, \varepsilon) \rightarrow a > 0$, as $\varepsilon \rightarrow 0$.
  - Stochastically stable networks must be pairwise stable networks or networks of closed cycles
  - Stochastic stability identifies the most “robust” or easy to reach networks in a particular sense (the most mutations needed to get “unstuck”).
  - The above example converges to a Pareto efficient pairwise stable network by considering all the possible dynamic paths between the left and right networks.

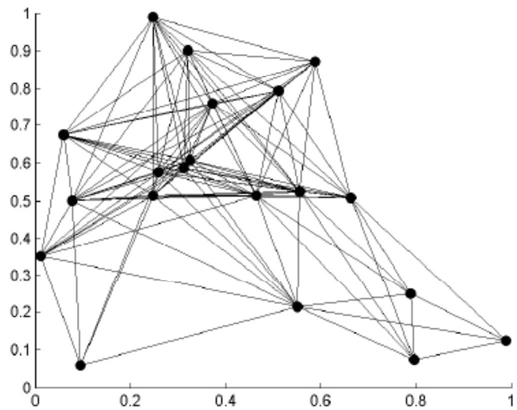
Coalition Formation at the Stable State

- The cost depends on the physical locations of nodes
  - Random network where nodes are placed according to a uniform Poisson point process on the [0,1] x [0,1] square.
- Theorem: The coalition formation at the stable state for n→∞
  - Given $\delta = 0,\quad V = P\left(\frac{\ln n}{n}\right)^{\frac{\alpha}{2}}$ is a sharp threshold for establishing the grand coalition (number of coalitions = 1).
  - For $0 < \delta \leq 1$, the threshold is less than $P\left(\frac{\ln n}{n}\right)^{\frac{\alpha}{2}}$.

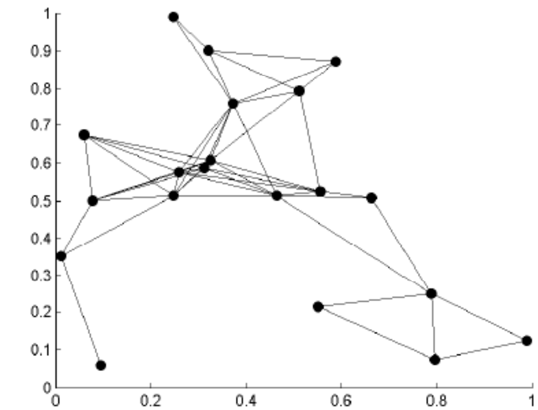


n = 20

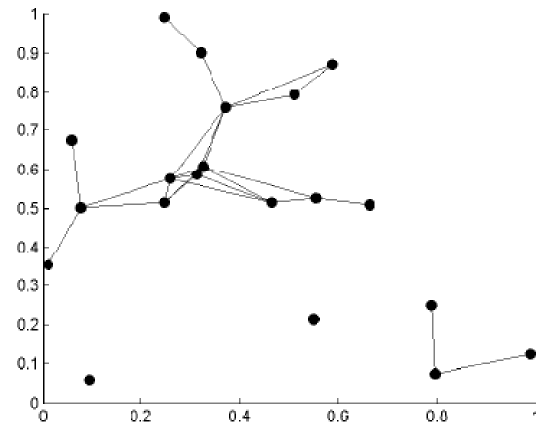
Topologies Formed



(a) P = 0.5 (low cost); complete graph



(b) P = 2 (middle cost); small world topology



(c) P = 4 (high cost); partitioned network

Time-dependent Game

- The game is time-dependent
  - The payoff players receive **varies over time**.
  - The dynamics of the game can be separated in rounds of successive coalition expansions (or contractions).

- The dynamic coalition formation process is described as an iterated game
  - x_i^t : the action *i* chooses at time *t*.
  - $v_i\left(x^t\right)$: the payoff of user *i* at time *t.*
  - $q^t(x)$: players' probability of playing action *x* at time *t.*
  - C_i^t : the set of users that form the coalition user *i* belongs to at time *t*.
  - user *i* and user *j* decide to activate link *ij* at time *t:*

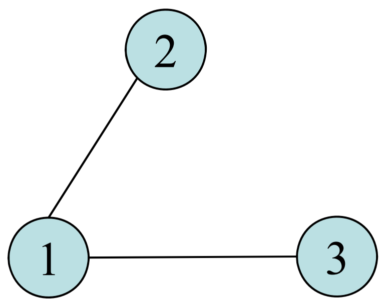
$$C_i^t = C_j^t = C_i^{t-1} \bigcup C_j^{t-1}$$

Value Function

- Value function for coalition *C* (component-wise additive value function)

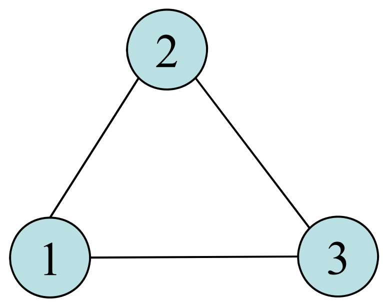
$$v(C) = \sum_{i \in C} v_i(g)$$

- Value function depends on topology



Same coalition $C=\{1,2,3\}$
with different topology

$v(\{12,13\}) \neq v(\{12,23,13\})$



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Networks and Trust

- **Trust and reputation critical for collaboration**
- Characteristics of trust relations:
  - *Integrative* (Parsons1937) – main source of social order
  - *Reduction of complexity* – without it bureaucracy and transaction complexity increases (Luhmann 1988)
  - *Trust as a lubricant for cooperation* (Arrow 1974) – rational choice theory
- **Social Webs, Economic Webs**
  - MySpace, Facebook, Windows Live Spaces, Flickr, Classmates Online, Orkut, Yahoo! Groups, MSN Groups
  - e-commerce, e-XYZ, services and service composition
  - **Reputation** and **recommender** systems

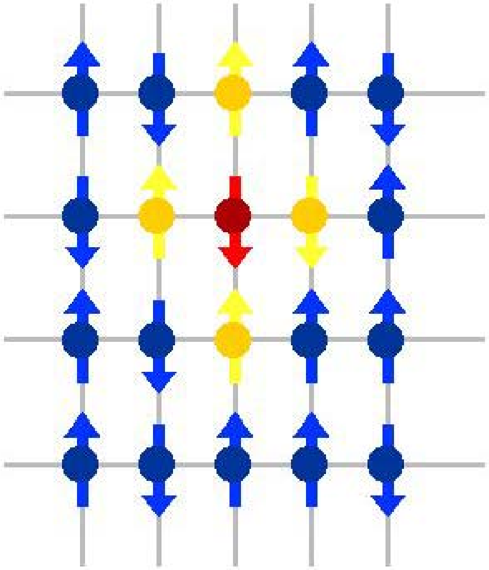
Ising and Spin Glass Models

- Statistical Physics models for magnetization

  Orientation of each particle's spin depends on its **neighbors**

  Ising Model: behavior of simple magnets

  Spin Glass Model: complex materials



- Interpretation:

  $\boldsymbol{s} = \{s_1, s_2, \ldots, s_n\}$ is a configuration of n particle spins -- $s_j = 1$ or -1 (up or down)

  Energy for configuration $\boldsymbol{s}$

$$H(\mathbf{s}) = -\frac{1}{T}\sum_{\substack{i \in V \\ j \in N_i}} J_{ij} s_i s_j - \frac{mH}{T}\sum_i s_i$$

  – **Ising Model**: $J_{ij} = J$ for all i, j

  – **Spin Glass Model**: J_{ij} depend on i,j and can be **random**

Ising/SG Models and Games

- Ising/SG models can be interpreted as dynamic (repeated) games:
  - The value of s_i represents whether node i is willing to cooperate or not
  - each particle selects spin to **maximize** its own *payoff*

$$\pi_i = \left(\sum_{j \in N_i} J_{ij} s_i s_j\right) / T$$

  - Ising model ($J_{ij} = J > 0$) : align its spin with the majority of neighbors spin
    - High T, conservative agents, not willing to change, small payoffs
    - Low T, aggressive agents, larger payoffs
  - Collection of local decisions reduces the total energy of the interacting particles

- Inspires an approach where **trust** is an **incentive for cooperation**
  - J_{ij} can be interpreted as the **worth of player *j* to player *i***
  - decide to cooperate or not based on benefit from cooperation **and** trust values of neighbors

Trust as Mechanism to Induce Collaboration

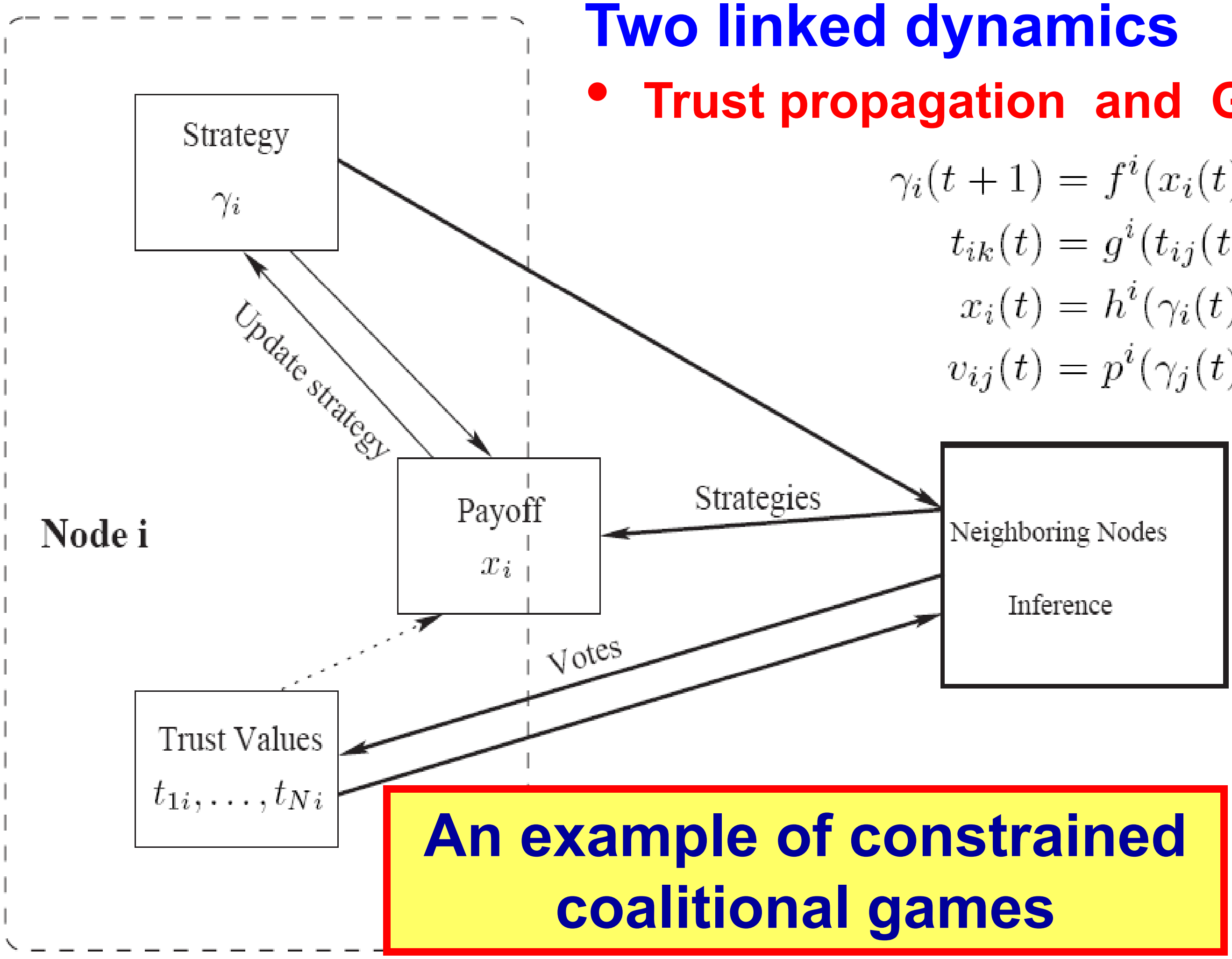
- Trust **is an incentive** for collaboration
  - Nodes who refrain from cooperation get lower trust values
  - Eventually penalized because **other nodes tend to only cooperate with highly trusted ones**.
- For node i **loss for not cooperating** with node j is a nondecreasing function of J_{ji}, $f(J_{ji})$,
- New characteristic function is

$$\nu(\mathcal{S}) = \sum_{i,j\in\mathcal{S}} J_{ij} - \sum_{i\in\mathcal{S},j\notin\mathcal{S}} f(J_{ij})$$

- **Theorem** : if $\forall i, j,\ J_{ij} + f(J_{ji}) \geq 0$, the core is nonempty and $x_i = \sum_{j\in N_i} J_{ij}$ is a feasible payoff allocation in the core.

By introducing a trust mechanism, all nodes are induced to collaborate without any negotiation

Dynamic Coalition Formation



Two linked dynamics

- **Trust propagation and Game evolution**

$$\gamma_i(t+1) = f^i(x_i(t), \gamma_i(t), \gamma_j(t), t_{ij}(t))$$
$$t_{ik}(t) = g^i(t_{ij}(t), v_{jk}(t)) \quad \forall k \in N$$
$$x_i(t) = h^i(\gamma_i(t), \gamma_j(t))$$
$$v_{ij}(t) = p^i(\gamma_j(t), t_{ji}(t))$$

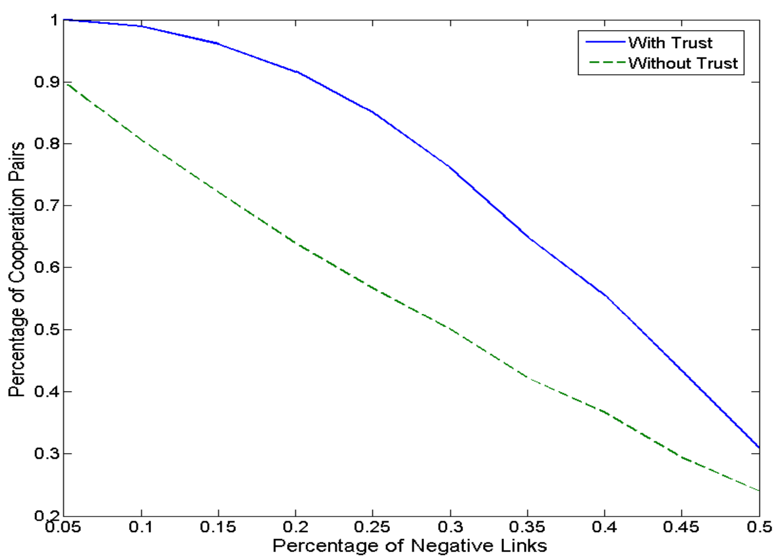
An example of constrained coalitional games

Stability of dynamic coalition

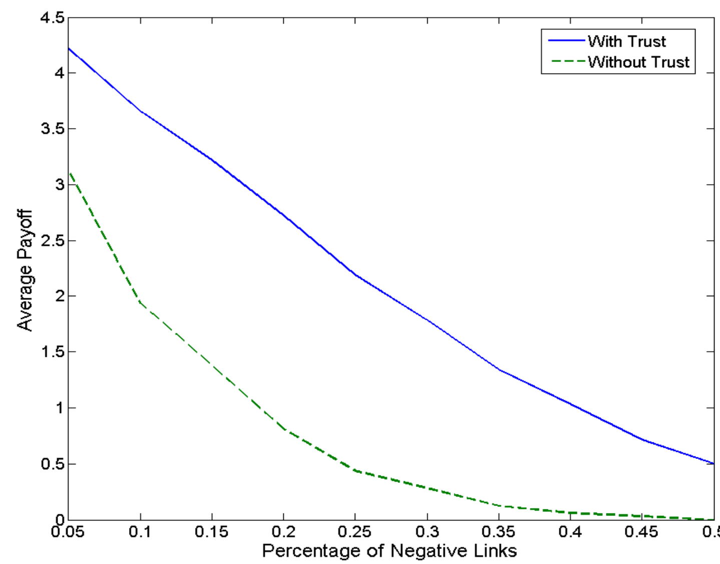
Nash equilibrium

Results of Game Evolution

- **Theorem**: $\forall i \in N_i$ and $x_i = \sum_{j \in N_i} J_{ij}$, there exists τ_0, such that for a reestablishing period $\tau > \tau_0$
  - terated game converges to Nash equilibrium;
  - In the Nash equilibrium, all nodes cooperate with all their neighbors.
- Compare games with (without) trust mechanism, strategy update:



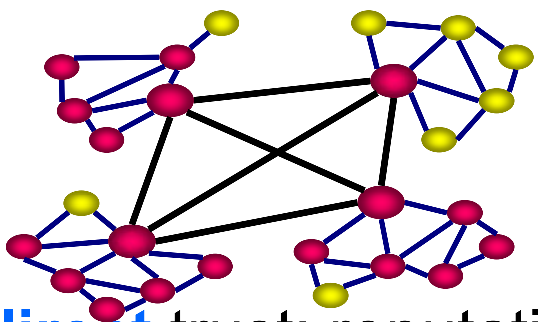
Percentage of cooperating pairs *vs* negative links



Average payoffs *vs* negative links

Next Generation Trust Analytics

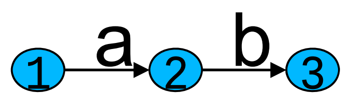
- Trust evaluation, trust and mistrust dynamics
  - Spin glasses (from **statistical physics**), phase transitions



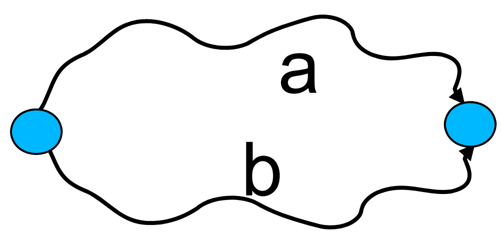
$$s_i(k+1) = f\left(\hat{J}_{ji}, s_j(k) \mid j \in N_i\right)$$

- **Indirect** trust; reputations, profiles; Trust computation via 'linear' **iterations in ordered semirings**

$$a \otimes b \leq a, b$$



$$a \oplus b \geq a, b$$



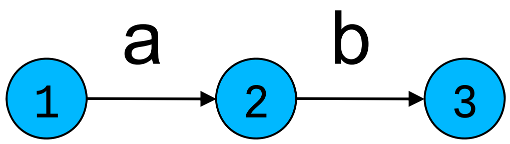
2007 IEEE Leonard Abraham prize
New Book **"Path Problems in Networks" 2010**

- **Direct trust: Iterated pairwise games on graphs** with players of many types

Generalized Networks and Semirings

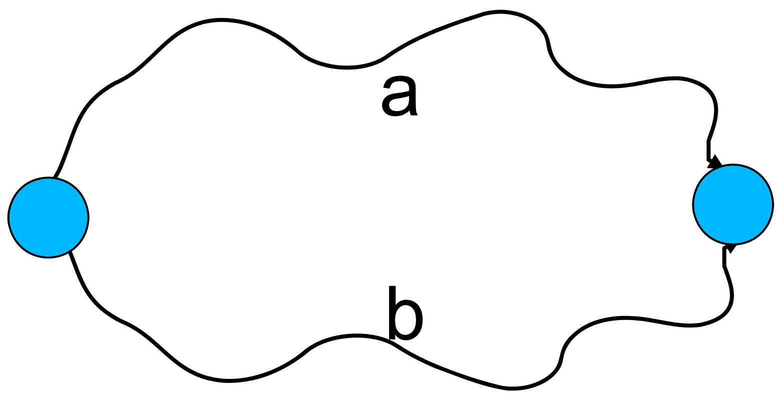
- Combined **along-a-path weight should not increase** :

$$a \otimes b \leq a, b$$



- Combined **across-paths weight should not decrease** :

$$a \oplus b \geq a, b$$



Distributed Weight Dynamics

- Path interpretation

$$t_{i \to j} = \bigoplus_{\text{path } p: i \to j} t^{p}_{i \to j}$$

- **Linear system interpretation**

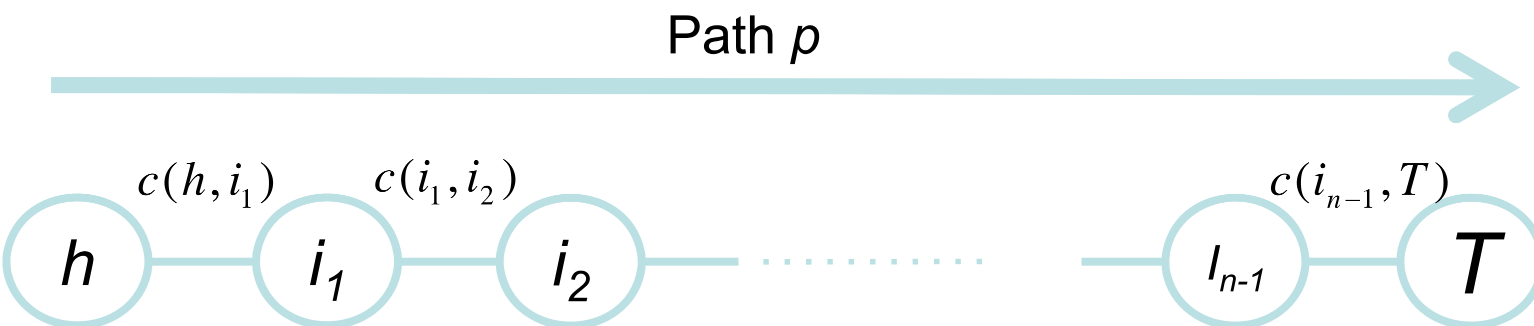
$$t_{i \to j} = \bigoplus_{User\ k} t_{i \to k} \oplus w_{k \to j}$$

$$\vec{t}_n = W \otimes \vec{t}_{n-1} \oplus \vec{b}$$

- Treat as a **linear system**
  – We are looking for its **steady state**.
- Have resolved also **shortest path sensitivity** in semiring framework

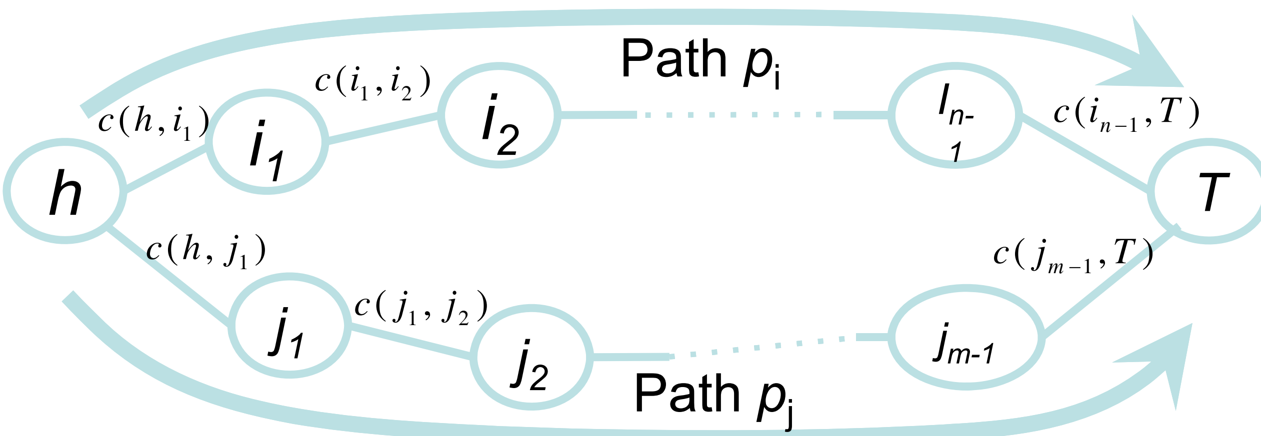
Graphs and Semirings

Consider undirected graph $G(V,E)$ with edge weights $c(i,j) \in S$.



Weight of the path p by edge composition is

$$w(p) = c(h,i_1) \otimes c(i_1,i_2) \otimes \ldots \otimes c(i_{n-1},T)$$



Composing path weights:

$$w(p_i) \oplus w(p_j)$$

Ordered Semirings

- Functions with semiring structure lend themselves to distributed computation/evaluation
- **Ordered semirings** $\oplus$ is the supremum or infimum operator and $(S, \otimes, \preceq)$ is an ordered semigroup
- Associate with every edge (*i*, *j*) of the dynamic graph a semiring element $c(i,j)[t] \in S$
- **General semiring optimal path problem** on a dynamic graph corresponds to computing

$$p^*(S,T)[t] = \arg\oplus_{p \in P_{S,T}[t]} \otimes_{(i,j)\in p} c(i,j)[t]$$

Multi-Criteria Shortest Path (several metrics)

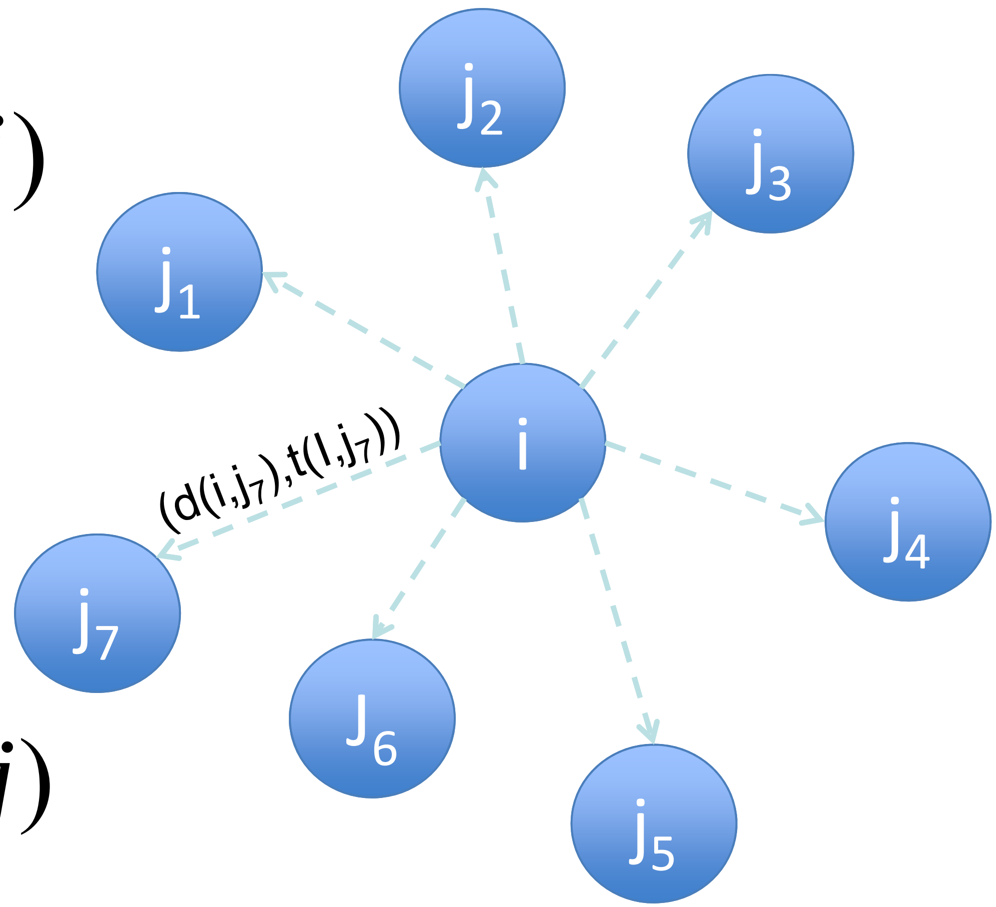
- **D** of a path "p"

$$d(p) = \sum_{(i,j)\in p} d(i,j)$$

- **T** of a path "p" – bottleneck

$$t(p) = \min_{(i,j)\in p} t(i,j)$$

Bi-metric Network



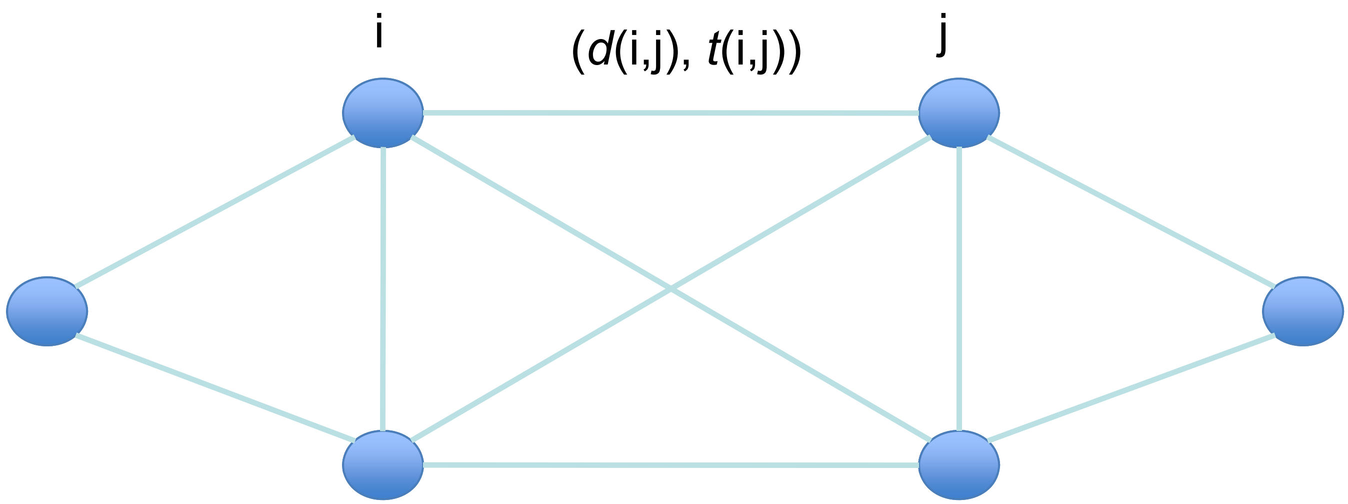
MCOP

- The two metrics are not trivially comparable.

$$MCOP: \quad (\mathcal{P}_{SD}, f, X) / \theta / (\mathcal{R}^{Q}, \preceq)$$

Notation	Definition	Name
$x \leq y$	$x_i \leq y_i \quad i = 1,2,..,Q$	Weak component-wise order
$x < y$	$x_i \leq y_i \quad i = 1,2,..,Q$ and $x \neq y$	Component-wise order
$x \ll y$	$x_i < y_i \quad i = 1,2,..,Q$	Strict component-wise order
$x \leq_{lex} y$	$x_k < y_k \quad or \quad x = y \quad k = \min\{i : x_i \neq y_i\}$	Lexicographic component-wise order
$x \leq_{MO} y$	$\max_i x_i \leq \max_i y_i$	Max order

Path problems on Graphs – D and T Semirings



$$\min_{p \in \mathcal{P}_{SD}} d(p) = \min_{p \in \mathcal{P}_{SD}} \sum_{(i,j) \in p} d(i,j)$$

$$\max_{p \in \mathcal{P}_{SD}} t(p) = \max_{p \in \mathcal{P}_{SD}} \left(\min_{(i,j) \in p} t(i,j) \right) = -\min_{p \in \mathcal{P}_{SD}} \left(\max_{(i,j) \in p} (-t(i,j)) \right)$$

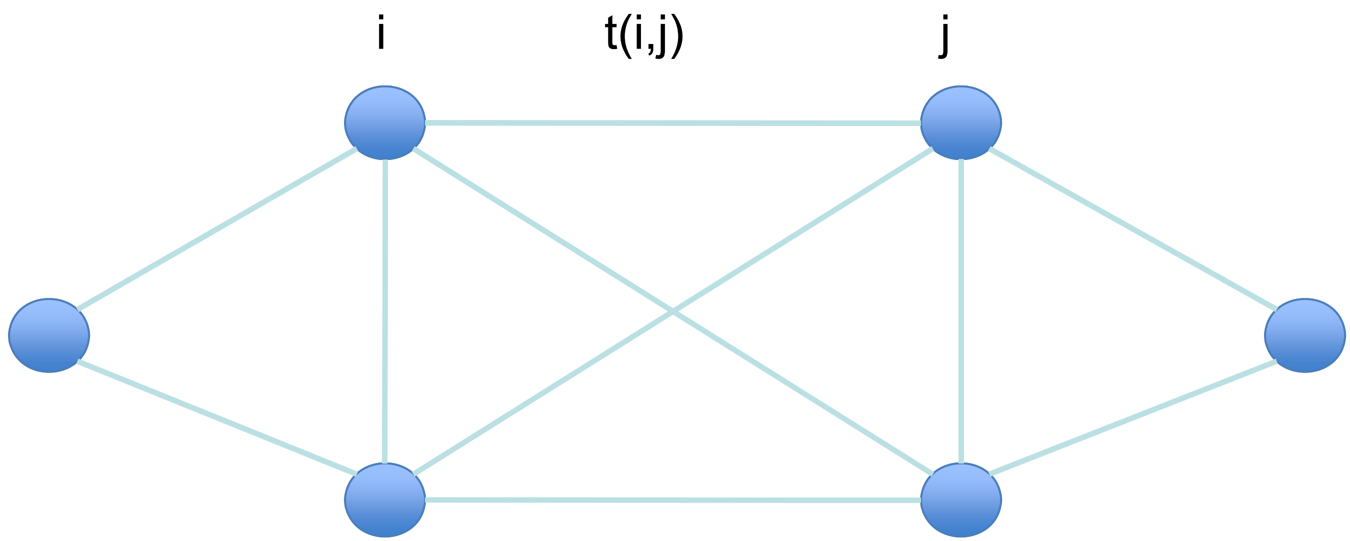
D Semiring: $(\mathcal{R}_+ \cup \{0\},\ \min,\ +)$

T Semiring: $(-\mathcal{R}_+ \cup \{0\},\ \min,\ \max)$

$$f : \mathcal{P}_{SD} \to R^2$$
$$f(p) = (d(p), -t(p)),\ \forall p \in \mathcal{P}_{SD}$$

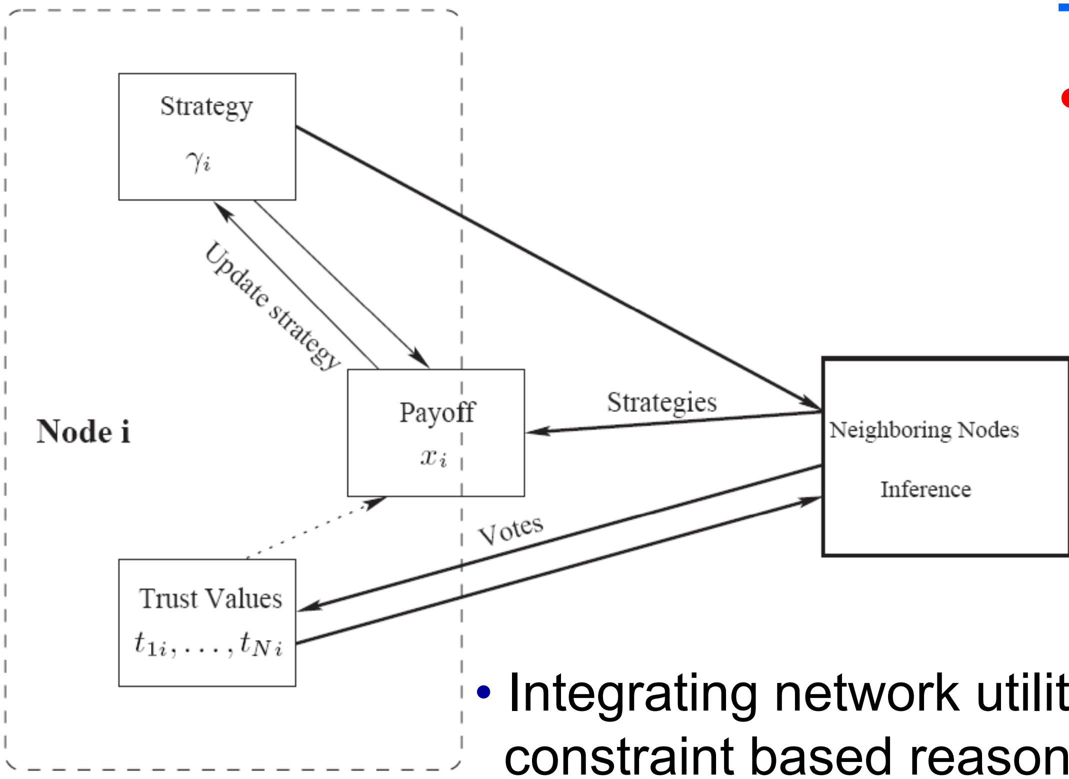
Notions of Optimality: Pareto, Lexicographic, Max-Ordering, Approx. Semirings

Pareto optimal paths – Edge exclusion algorithm



- Edge exclusion – From $G(V,E)$, remove all the edges whose $t(i,j) > \varepsilon$ to obtain a graph $G'(\varepsilon)$
- $G'(\varepsilon)$ contains paths which have all $t(i,j) \leq \varepsilon$
- We can also show that G' has all paths in G which have $t(i,j) \leq \varepsilon$ and only those

Constrained Coalitional Games: Trust and Collaboration



Two linked dynamics

- Trust / Reputation propagation and Game evolution

- Integrating network utility maximization (NUM) with constraint based reasoning and coalitional games

- Beyond linear algebra and weights, semirings of constraints, constraint programming, soft constraints semirings, policies, agents
- Learning on graphs and network dynamic games: behavior, adversaries
- Adversarial models, attacks, constrained shortest paths, …

Outline

- Multiple interacting dynamic hypergraphs – four challenges
- Networks and Collaboration -- Constrained Coalitional Games
- Trust and Networks
- Component-based network synthesis
- Topology and performance
- New probability models (non Kolmogorov)
- Biological networks and cancer dynamics
- Conclusions and Future Directions

Component-Based Heterogeneous Network Synthesis

- **How to synthesize resilient, robust, adaptive networks?**

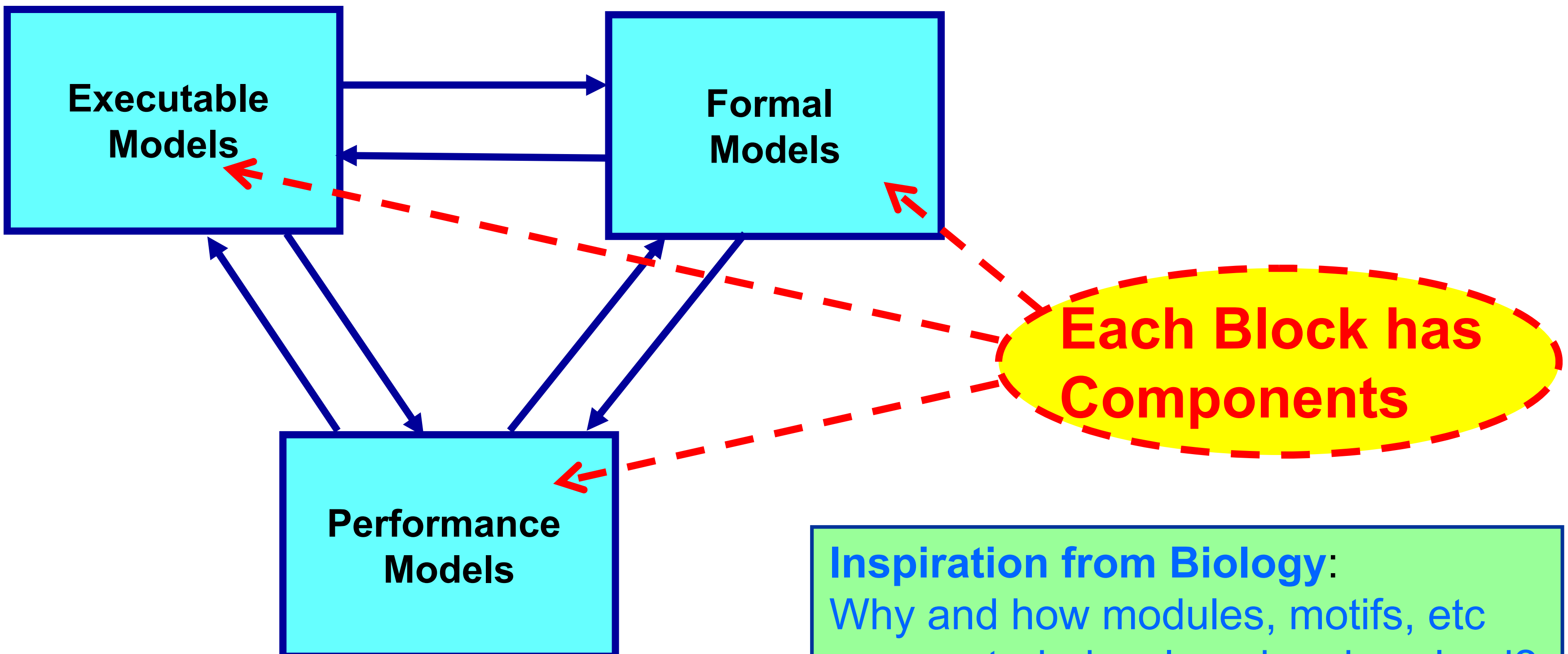
  → ***Component-Based Network Analysis & Synthesis (CBN)***
- **Components: modularity, cost reduction, re - usability, adaptability to goals, new technology insertion, validation and verification**
- **Interfaces: richer functionality– intelligent/cognitive networks**
- **Theory and Practice of Component-Based Networks**
  - **Heterogeneous components and compositionality**
  - **Performance of components and of their compositions**
  - **Back and forth from performance - optimization domain to correctness and timing analysis domain and have composition theory preserving component properties as you try to satisfy specs in both domains**
- **From communication to social, from cellular to transportation, from nano to macro networks**
- **Critical theory and methodology for Networked Embedded Systems, CyBer Physical Systems, Systems Biology**

Networks: Different Linked Views

Networks:

- as **distributed, asynchronous, feedback (many loops), hybrid automata (dynamical systems)**
- as **distributed asynchronous active databases** and **knowledge bases**
- as **distributed asynchronous computers**

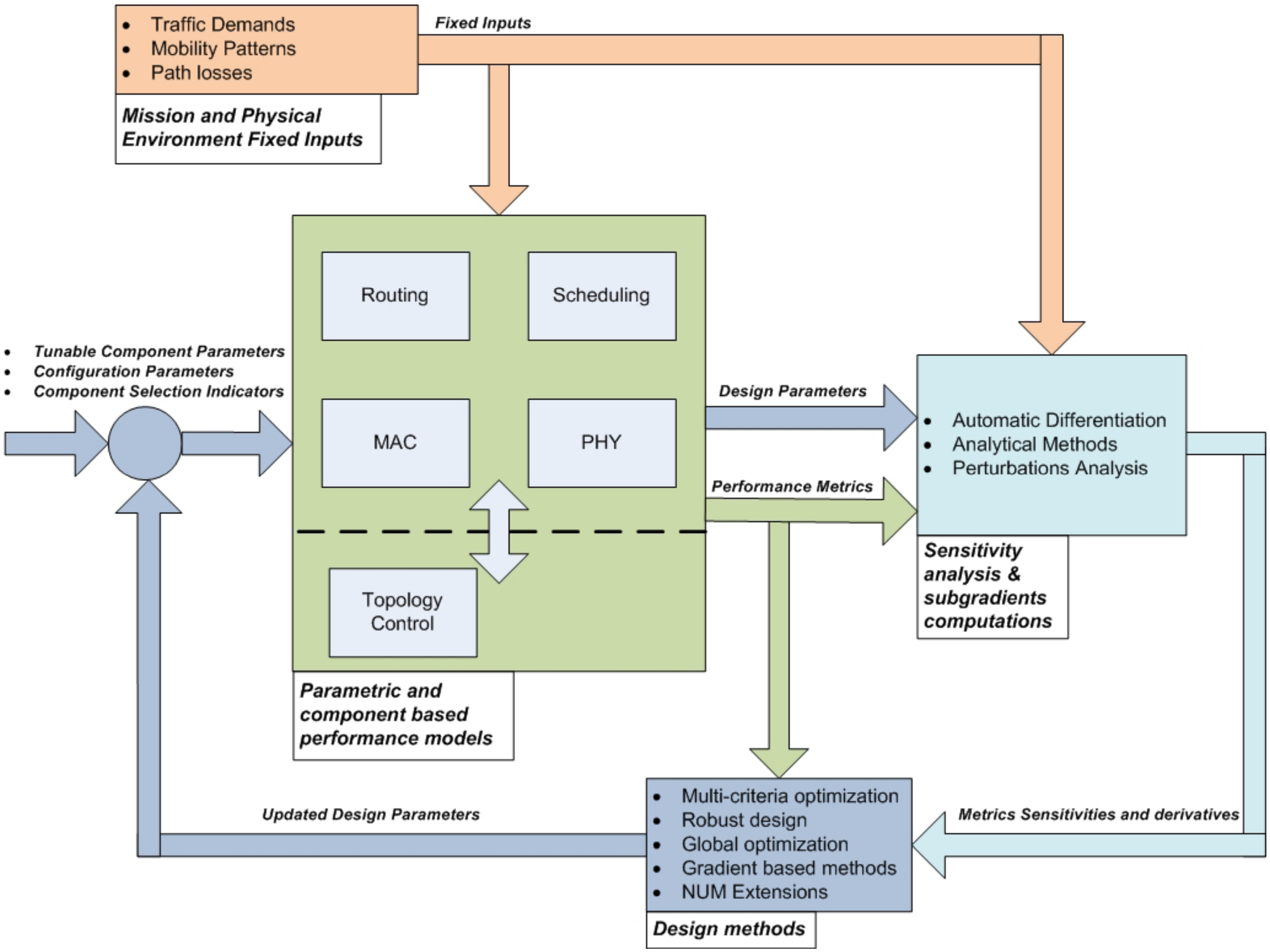
Component-Based Heterogeneous Networked Systems Synthesis



Inspiration from Biology: Why and how modules, motifs, etc are created, developed and evolved?

Grand challenge: Develop this framework for distributed, partially asynchronous systems, with heterogeneous components and time semantics

Component Based Routing (CBR) and Networking (CBN) for MANET



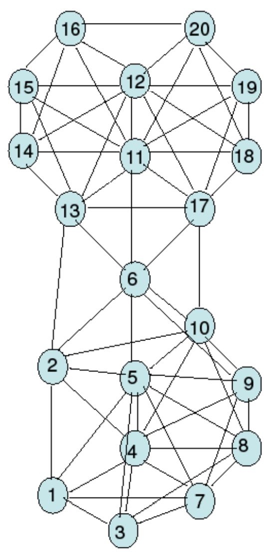
MAC and Routing Components

- **Objective**
  - Design MANET adaptable to missions with predictable performance
- **Approach**
  - **Break traditional layers to components!** Develop component-based models MANET that considers cross-layer dependency to improve the performance
  - Study the effect of each component on the overall MANET performance

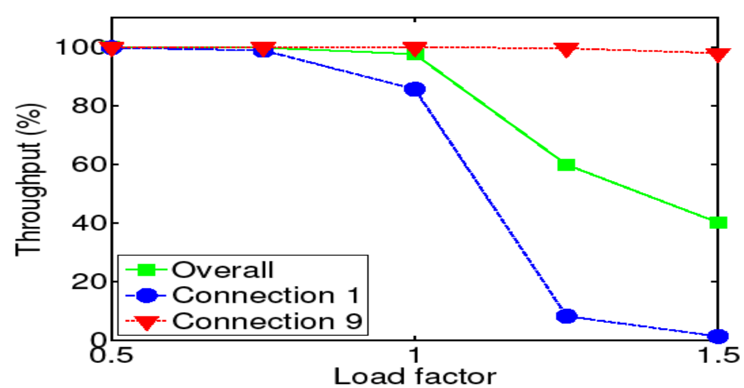
- **Routing Components** – routing protocols like OLSR [Baras08]
  - Neighbor Discovery Component (NDC)
  - Selector of Topology Information to Disseminate Component (STIDC)
  - Topology dissemination Component (TDC)
  - Route Selection Component (RSC)
- **MAC Components** – based on CSMA-CA MAC protocols like IEEE 802.11 [Baras08], and on schedules based MAC (USAP) [Baras09]
  - Scheduler
  - MAC

Results Todate

- Realistic MANET scenarios from DARPA CB-MANET benchmarks
- 20 node, 10 connections -- up to 50 moving nodes with disconnections (ground and UAVs)
- **Substantial improvements of performance through new NDC and STIDC** components (as compared to traditional OLSR) – being reported to MANET WG of IRTF



PHY Layer Connectivity



Throughputs for increasing load

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Distributed Algorithms in Networked Systems and Topologies

- Distributed algorithms are essential
  - Group of agents with certain abilities
  - Agents **communicate with neighbors,** share/process information
  - Agents **perform local** actions
  - **Emergence** of global behaviors
- **Effectiveness** of distributed algorithms
  - The **speed** of convergence
  - **Robustness** to agent/connection failures
  - Energy/ communication **efficiency**
- **Group topology affects** group performance
- **Design problem**:

  Find graph topologies with favorable tradeoff between performance improvement (**benefit**) *vs* **cost** of collaboration
- **Example: Small Word graphs** in consensus problems

Design of information flow

Consensus problems

$$x(k+1) = F(k)x(k)$$

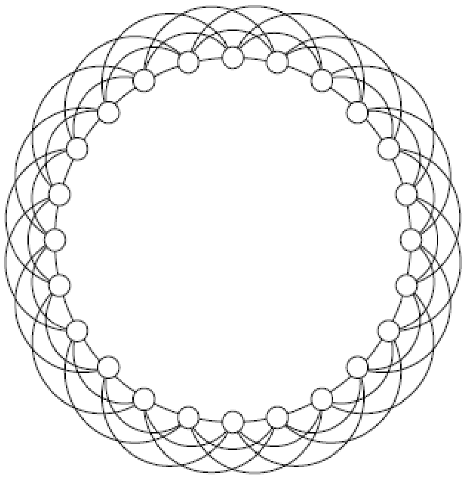
$$F(k) = (I + D(k))^{-1}(A(k) + I)$$

$$F(k) = I - hL(k)$$

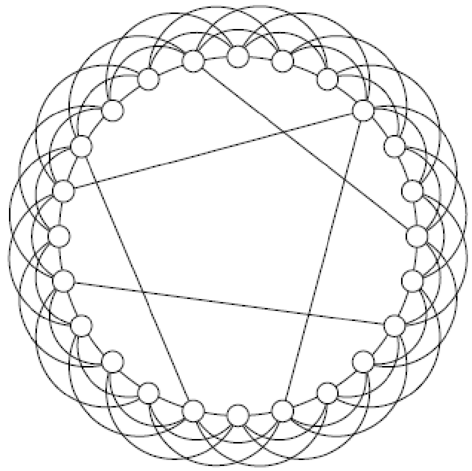
Symmetric communication

- **Fixed graphs**: Geometric convergence with rate equal to Second Largest Eigenvalue Modulus (SLEM)
- How does **graph topology affect** location of eigenvalues?
- How can we **design graph topologies** which result in good convergence speed?

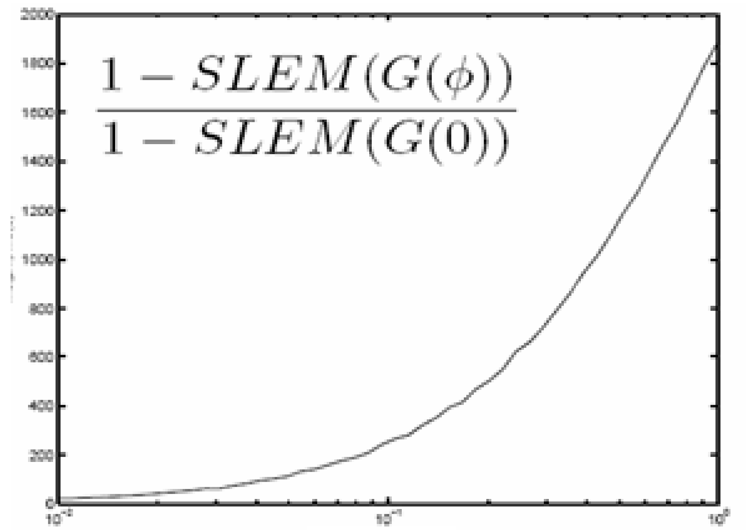
Small World Graphs



Simple Lattice
C(n,k)



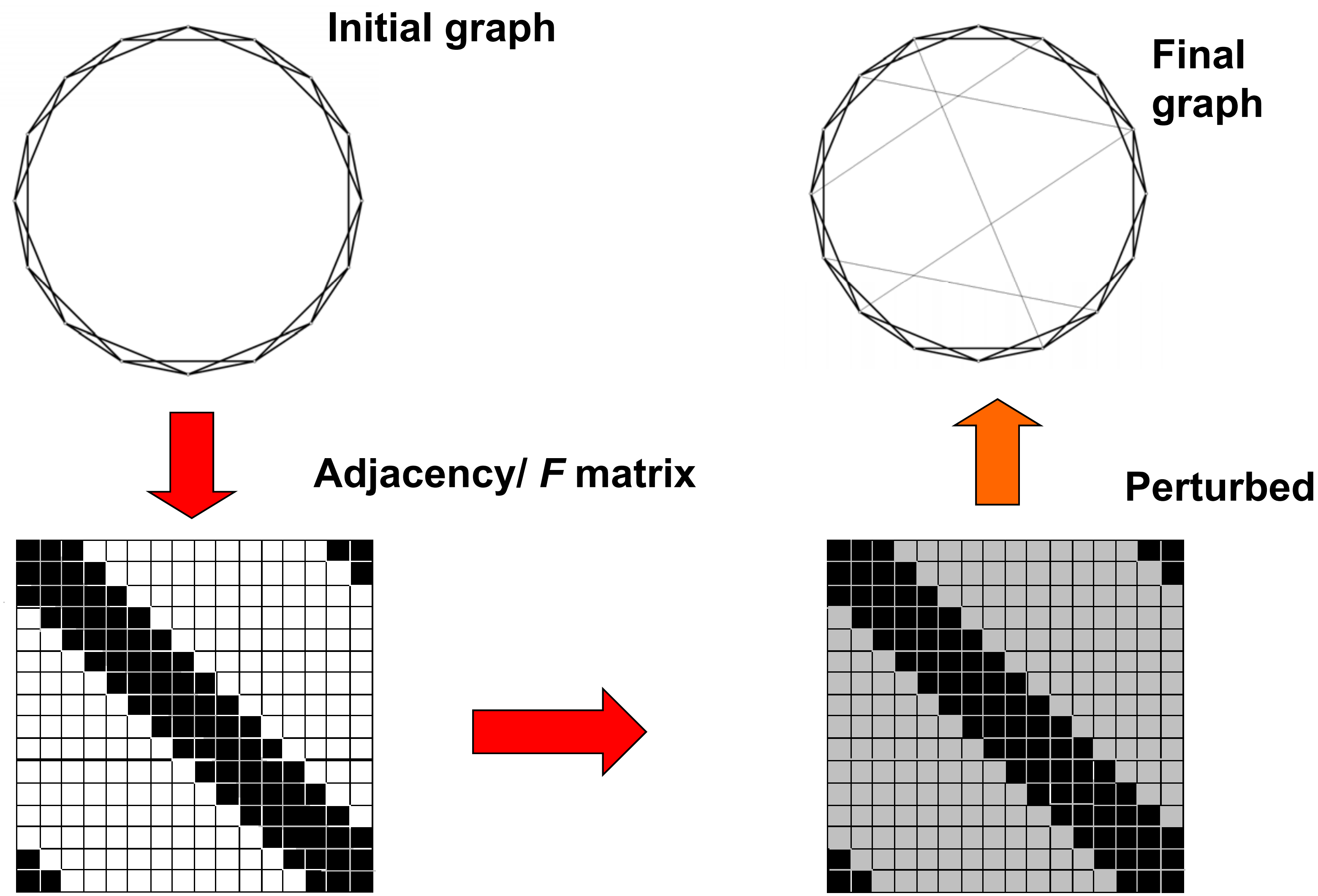
Small world: Slight variation adding $nk\Phi$



\phi
C(1000,5)

Adding a **small portion** of well-chosen links → **significant increase** in convergence rate

Mean Field Explanation and Perturbation Approach



Watts-Strogatz Small World networks

- Random graph approach
  (e.g. Durrett 2007, Tahbaz and Jadbabaie 2007)
- **Perturbation** approach (Higham 2003)
  - Start from lattice structure $G_0 = C(n,k) \longleftrightarrow F_0$
  - Perturb zero elements in the positive direction by $\varepsilon = \frac{K}{n^{\alpha}}$ for fixed $K > 0$ and $\alpha > 1$.
  - Perturb the formerly nonzero elements equally, such that the stochastic structure of the *F* matrix is preserved F_{ε}
  - Analyze the SLEM as a function of the perturbation as α varies

1- D case …

- Refer to the perturbations as ***ε*-shortcuts**
- In the limit of large *n* :
  - For $\alpha > 3$ the effect of *ε*-shortcuts on convergence rate is negligible
  - For $\alpha = 3$ the effect of ε-shortcuts on convergence rate starts (spectral gap gain perturbation of same order)
  - **For $\alpha = 2$ the shortcuts dominantly decrease SLEM**
  - For $\alpha = 1$ SLEM is very small
- *ε*-shortcuts are loosely analogous to the shortcuts in Small World networks
- ***α* = *3*** can be considered as the onset of small world effect with **small world effect happening** at ***α* = 2**

Distributed exploration of the graph structure

- **Self-organization for better performance and resiliency**
- Hierarchical scheme to design a network structure capable of running **distributed algorithms with high convergence speed**
- A two stage algorithm:

  1- Find the most effective choice of **local leaders**

  2- Provide nodes with information about their location **with respect to other nodes and leaders** and the choice of groups to form
- Divide N agents into K groups with M members each

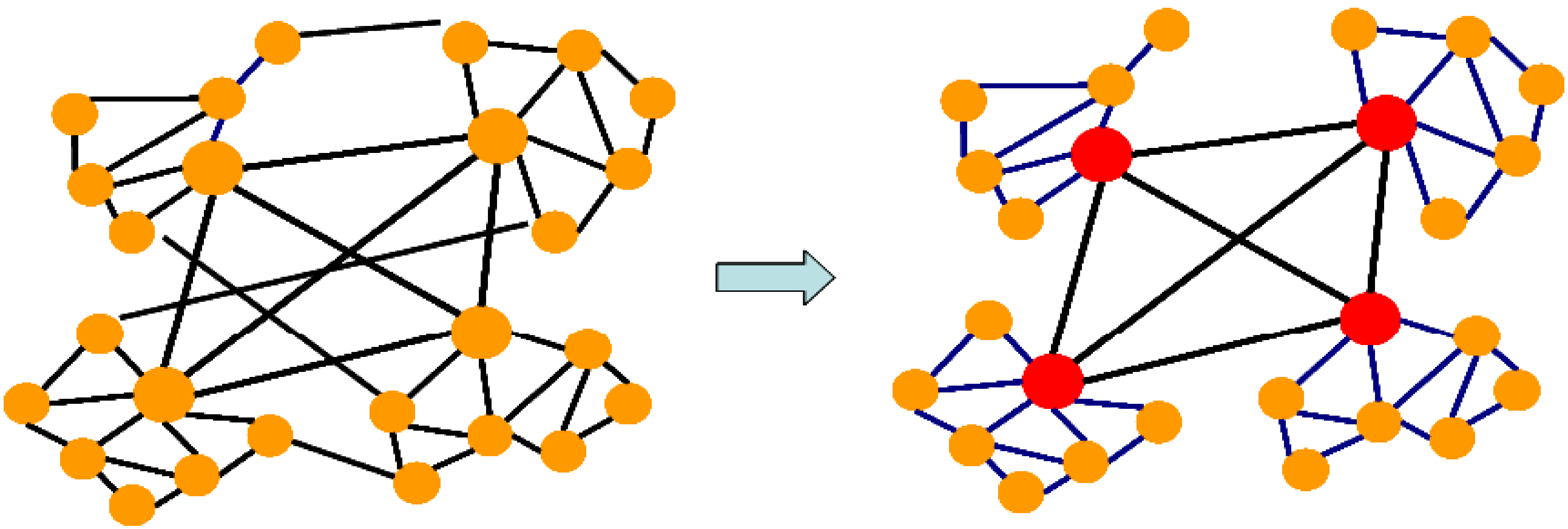
  $N = K \times M, \quad K \leq M \times N$, select 'leaders'

Distributed self - organization

Determine leaders → Find influence → Remove unnecessary links

Semi-decentralized

Decentralized: Dirichlet problem on the graph



Goal: design a scheme that gives each node a vector of compact global information

Two stage semi-decentralized algorithm

- **Stage 1: Determining *K* leaders**
  - Each node determines its social degree via local query
  - Dominant nodes in each neighborhood send their degrees to the central authority
  - Central authority computes their social scores

$$SC(k) = \alpha SD^{(2)}(k) + (1-\alpha) SD^{(3)}(k)$$

    Choice of α determines whether leaders in star-like neighborhoods are preferred

  - The central authority selects the K nodes with highest scores as social leaders and gives them an arbitrary order

Algorithm (cont'd)

- **Stage 2**: **Determining the influence vectors**
  - Based on its order each leader takes its influence vector to be the fixed vector e_i
  - Regular nodes update their influence vector entries:

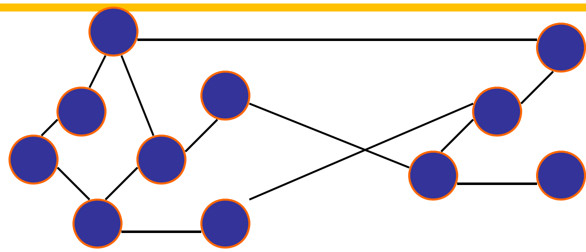
$$x_i^k(t+1) = \frac{1}{n_i + 1}\left[x_i^k(t) + \sum_{j \in N_i(t)} x_j^k(t) \right]$$

- For connected graphs, for t large enough, x_i^k converges to the influence of leader *k* on node *I*

- Upon calculation of influence vectors, each regular node determines its local leader and stops its communication with neighbors who have other leaders

- **Graph decomposes into two level hierarchy with efficient communication pattern**

Reliability and Spanning Trees



$\mathcal{G}(\mathcal{V},\mathcal{E})$

$\mathcal{V} = \{1, 2, ..., n\}$

$\mathcal{E} = \{l_1, l_2, ..., l_e\}$

p : Constant link loss probability

N_i : # of connected components with i edges

$\tau(\mathcal{G})$: Number of spanning trees

$$\mathrm{Rel}(\mathcal{G}, p) = \sum_{i=n-1}^{e} \mathrm{N}_i (1-p)^i p^{e-i}$$

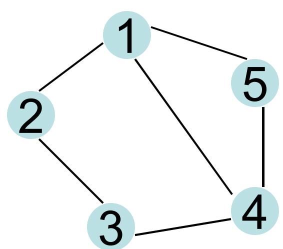
For sufficiently large p:

$$\tau(\mathcal{G})(1-p)^{n-1} p^{e-n+1} \leq \mathrm{Rel}(\mathcal{G}, p) \leq \tau(\mathcal{G})(1-p)^{n-1}$$

- End to end applications
- Spanning tree as a minimally connected graph
- *T(G)* as a measure of robustness to losses
- *References*: Kelmans, Colbourn

Graph Theory for Robust Network Design

- Goal: Given a base topology add *k* edges from a set of *m* candidates such that results in maximum number of spanning trees
- Number of spanning trees $\tau(G) = \frac{1}{n}\prod_{i=2}^{n}\lambda_i(L) = \frac{1}{n}\det(L + \frac{11^T}{n})$
- Incidence vector of an edge shows between which nodes it is



$$f_i : \text{incidence vector}$$
$$f_i = e_\alpha - e_\beta$$
$$f_{(1,5)} = e_1 - e_5 = [1 \quad 0 \quad 0 \quad 0 \quad -1]^T$$

$$f_{15} f_{15}^T = \begin{pmatrix} 1 & 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & 1 \end{pmatrix}$$

- Graph Laplacian $L = D - A = F_{nm}F_{mn}^T = \sum_{i=1}^{m} f_i f_i^T$

Problem Statement

- Goal: Given a base topology add *k* edges from a set of *m* candidates such that results in maximum number of spanning trees $$\tau(\mathcal{G}) = \frac{1}{n}\prod_{i=2}^{n}\lambda_i(L) = \frac{1}{n}\det(L + \frac{1 1^T}{n})$$
- Dynamic graph process resulting from adding edges

$$\text{Maximize } \tau(G(t+k))$$

Subject to:

$$\begin{cases} G(t+1) = Add(G(t), u(t)), & t = 0,1,\dots,k-1 \\ u(t) = e(t+1), & e(t+1) \in S \subseteq E(\bar{G}(t)) \\ G(t) = \mathcal{G}_0 & \end{cases}$$

Formulation and Relaxation

- Goal: Given a base topology add *k* edges from a set of *m* candidates such that results in maximum number of spanning trees (Approach similar to Ghosh and Boyd 06)

$$\text{Maximize} \quad \tau\left(L_0 + \sum_{i=1}^{m} x_i f_i f_i^T \right) \quad \text{or equivalently}$$

$$\log \det \left(L_0 + \frac{1}{n} J + \sum_{i=1}^{m} x_i f_i f_i^T \right)$$

$$\text{Subject to:}$$

$$1^T x = k$$

$$x \in \{0,1\}^m$$

$$\log \det \left(L_0 + \frac{1}{n} J + \sum_{i=1}^{m} x_i f_i f_i^T \right)$$

is concave in x.

- Relax to $x \geq 0$

$$x_i^* > 0 \Rightarrow \frac{\partial \tau(x^*)}{\partial x_i} \geq \frac{\partial \tau(x^*)}{\partial x_j}, \forall j$$

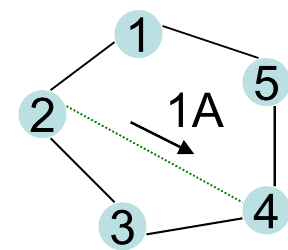
- At maximum $\tau(x)$ has equal derivatives for positive x_i s

Robust Network Design

- Derivative:

$$f_i^T \left(L_0 + \frac{1}{n} J + \sum_{i=1}^{m} x_i f_i f_i^T \right)^{-1} f_i = \lambda, \forall i \in \text{Chosen edge set } (\mathrm{x_i} > 0)$$

$$\mathrm{R_{eff}}(i) = f_i^T \left(L + \frac{1}{n} J \right)^{-1} f_i$$



$$R_{eff}(\alpha, \beta) = V_{\alpha\beta}$$

- If feasible, add edges such that the effective resistance distance of all selected edges become equal and greater than the effective resistance distance between non-selected candidates

Special Cases

- Adding 2 edges (α,β) and (γ,δ)

$$\tau(G(2)) = \Big[\big(1+R_{eff}(\alpha,\beta)\big)\big(1+R_{eff}(\gamma,\delta)\big) - \Big(\underbrace{(z_{\gamma\alpha}-z_{\gamma\beta})}-\underbrace{(z_{\delta\alpha}-z_{\delta\beta})}\Big)^2\Big]\tau(\mathcal{G}_0),$$

$$Z = [z_{ij}] = \left(L + \frac{1}{n}J\right)^{-1}$$

Maximized by adding edge between high resistance distance nodes

Maximized by adding edge to symmetrize the graph

- Adding 3 or more edges similar: more complex terms due to compromising between symmetrizing the graph and joining nodes with the highest resistance distance

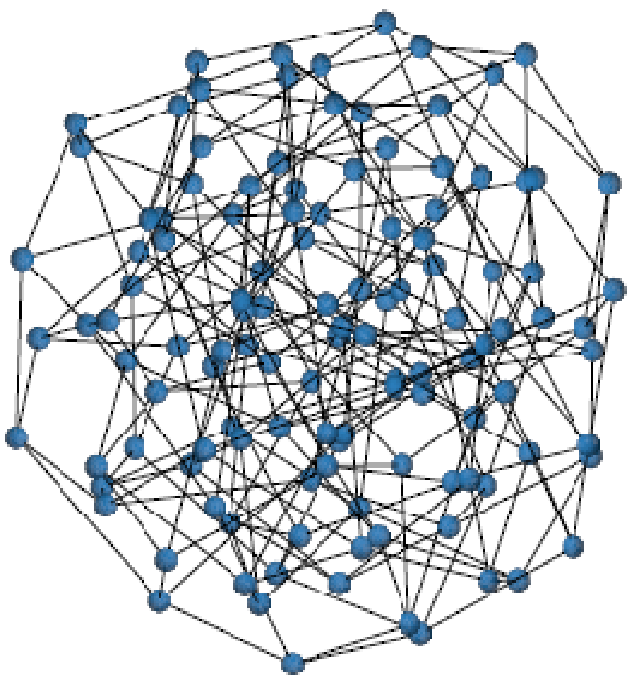
Expander Graphs

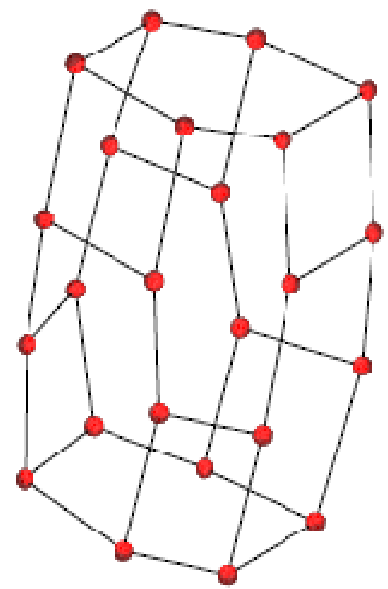
- Fast synchronization of a network of oscillators
- Network where any node is “nearby” any other
- Fast ‘diffusion’ of information in a network
- Fast convergence of consensus
- Decide connectivity with smallest memory
- Random walks converge rapidly
- Easy to construct, even in a distributed way (ZigZag graph product)
- Graph G, ***Cheeger constant h(G)***
  - All partitions of G to S and S^c ,

    $h(G)$=min (#edges connecting S and S^c) /

    (#nodes in smallest of S and S^c)
- (k , N, ε) **expander** : $h(G) > \varepsilon$; **sparse but locally well connected (1-*SLEM*(*G*) increases as $h(G)^2$)**

Expander Graphs – Ramanujan Graphs





Constructing Expander Graphs

- Possible methods:
  - Form a random expander as a $2d$-regular multi-graph in which the set of edges consists of d separate Hamiltonian cycles on APs (Law and Siu 2003)
  - Form a union of two spanning trees chosen independently from the uniform distribution over all spanning trees of a complete graph, implementable by a random walk method (Goyal et al. 2009)

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Non-Commutative Probability Models – New Logics

- **Key idea**: interaction between measurements by different agents and between system states and dynamics and measurements (Baras, 1977)
  - Now investigated vigorously in information retrieval systems (van Rijsbergen, 2004)
  - Asynchrony and concurrency
- **Key challenge**: understand the fundamentals of information collection and information flow in multi-agent stochastic control systems
- Witsenhausen's model of information patterns is not correct – even in its most general setting
- Need for new non-commutative probability models – **new logics** --projections in Hilbert space

The Setting and the Problems

- *N* agents, local states, local times
- Measurements and hypotheses supported and interpreted by local states
- Static problems: distributed detection and estimation problems
- Simple dynamics: like in information retrieval systems
- Complex system dynamics: full interactions between measurements and measurements and controls
- Must unify the probabilistic and logical aspects in a consistent way (see recent results of Abbes 2005 for probabilistic models over systems with concurrency constraints)

Models with Incompatibility Build-in

Theorem 5: *If (E, S, P, T) is an event-state-operation structure, then $(E, \leq, ')$ is an ortholattice. Moreover, if $p, q \epsilon E$, then $T_{p \wedge q} = (T_{p'} \tilde{\circ} T_q) \circ T_q$*

Theorem 6: *Assumptions as in Theorem 5 . Then for $p, q \epsilon E$ the following are equivalent:* (a) pCq, (b) $T_p \circ T_q = T_q \circ T_p$. *If pCq then $T_{p \wedge q} = T_p \circ T_q$.*

- **Active interpretation of operations**: can be thought of as a model for the combined operation of taking a measurement and applying a control law by the agent
- **Passive interpretation of operations**: system's interaction to measurements (used by recent results in information retrieval systems)
- We also get an interpretation of the **conjunction of incompatible events or measurements as "data fusion" or "agreement" between agents**

Final Results and Future

Theorem 7: *Databases and conditioning in a multi-agent stochastic control system can be used to construct an event-state-operation structure. This is a faithful representation.*

Theorem 8: *If $(E, \leq)$ is atomic and the atoms $\hat{E}$ are mapped to pure states under T, then (E, S, P, T) can be represented as the lattice of projections on a Hilbert space.*

- The **most useful 'practical' model**

  Finite dimensional Hilbert space, measurements to self-adjoint operators, states to trace one positive operators

- The **biggest payoff**:

  Our theory (extensions of above) allow the formulation of 'design' problems as convex problems over a pair of Banach spaces (one for measurements and one for controls)

- They also results **automatically to introduction of 'supervisors'**

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Biological Networks – Our Research

- Characterization of biological networks
- Discovery of elemental components (e.g. motifs) – Modular decomposition
- Network composition from modules
- Development of a taxonomy of network structure *vs* behavior *vs* biology
- Network dynamics and their interpretation
- Network inference and tomography
- Applications to disease pathology (e.g. cancer)
- Development of analytic/computational tools

Cancer and Systems Biology -- networks are key

- Emergent properties at the system level – not just at components
- Multiple-interacting networks – gene networks, protein networks, metabolic networks
- Network dynamics more important
- Complex diseases cause changes in network dynamics
- **Key Question**: detect these changes from repeated partial measurements (data) at different scales

Network Inference from Data

- Inference of network models, from data and prior structure knowledge
- Various types of dynamic networks: Networks of ODEs, Bayesian Networks, Boolean Networks, Hybrid Networks
- Stochastic Graph Processes, their representation and parameter/structure estimation, Time Varying MRF
- **Usage**: Network level analysis to improve cancer prognosis (e.g. metastasis in breast cancer via sub-networks of the protein-protein interaction network)
- Use alterations of the molecular network in malignant cells to identify oncogenes (e.g. for B-cell lymphomas)
  - Common alteration: upregulation of growth factor receptors (e.g. EGFR hyperactivated in several cancers

Use of Network Models and Associated Analysis in Cancer

- Develop multi-network models and use them to understand and detect crucial characteristics of cancerous cells:
- Independence from external growth signaling
- Insensitivity to antigrowth signaling and evasion of apoptosis
- Limitless replicative potential
- Sustained angiogenesis and metastasis
- **Example 1**: Use a human protein-protein interaction network model -- compute certain sub-networks – prove that they re good indicators of metastasis (used maximum mutual information and conditional likelihood classification) in breast cancer [Chuang et al, 2007]
- **Example 2**: Develop a network centric approach to distinguish normal from malignant cells and identify targets and effectors of specific biochemical perturbations , potentially useful for the identification of drug targets [Mani et al 2008]

Example: Dynamic Modularity in Protein Interaction Networks for Breast Cancer Prognosis

- [Taylor et al , 2009]
- Dynamic structure of human interactome can be used to predict patient outcome
- Identify inter-modular hub proteins co-expressed with their interacting partners and intra-modular hub proteins co-expressed with their interacting patterns
- Observed substantial differences in biochemical structure of two hub types
- Signaling domains found in inter-modular hub proteins – associated with oncogenesis
- Analysis with breast cancer patients – altered interactome useful as indicator of breast cancer prognosis

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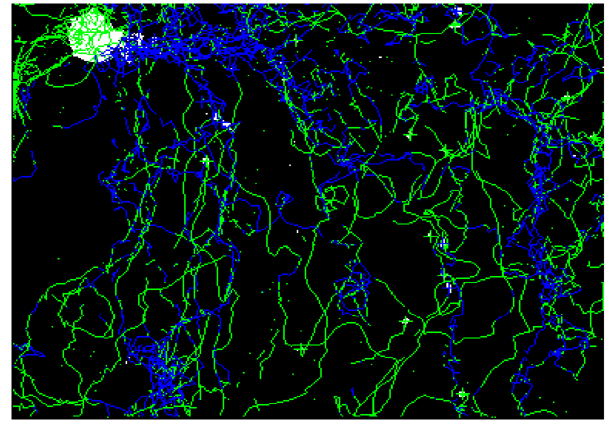
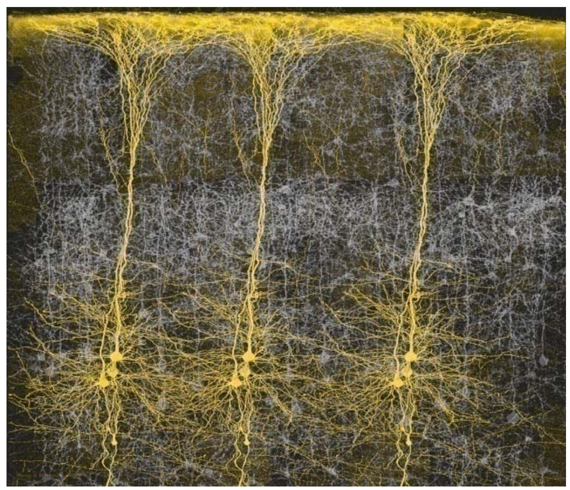
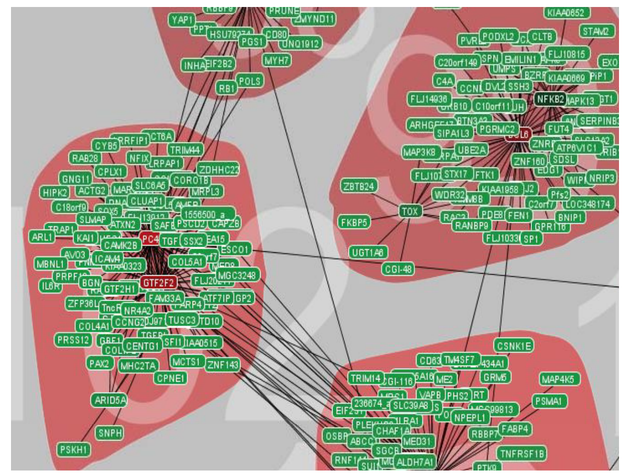
Conclusions

- Complex networks – multiple dynamic hypergraphs
- Fundamental tradeoff between the benefit from collaboration and the required cost for collaboration
- Coalitional games and network formation
- Trust as a catalyst for collaborations
- Component based network synthesis
- Effects of topology on distributed algorithm performance
- Performance vs. efficiency – small world graphs – expander graphs
- New probabilistic models (non-Kolmogorov)
- Biological networks and cancer dynamics

How Biology Does IT?







Lessons Learned -- Future Directions

- **Constrained coalitional games** – unifying concept
- Generalized networks, **flows - potentials**, duality and network optimization (monotropic optimization)
- **Time varying graphs** – mixing – statistical physics
- **Understand autonomy** – better to have self-organized topology capable of supporting (scalable, fast) a rich set of distributed algorithms (small world graphs, expander graphs) than optimized topology
- Given a set of distributed computations **is there a small set of simple rules** that when given to the nodes they can self-generate such topologies?

Thank you!

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Questions?