Surface wind stress model for turbulent flows above ocean surface waves
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Introduction
- About two-thirds of the surface of the Earth is covered by the ocean.
- The air-sea exchanges of mass, momentum, and energy over such a huge area play an integral role in determining the sea state, weather patterns, and climate and thus significantly impact many aspects of human life.
- Although we know that surface waves are critically important, we do not yet fully understand the fundamental physics of ocean waves.
- The current parameterizations of air-sea fluxes are limited, and that prevents us from, for example, making accurate predictions of extreme wind events such as tropical storms and hurricanes.
- The objective of this study is to develop an accurate wall-layer model for use in large-eddy simulations (LESs) that capture the salient features of air-sea fluxes.

Large-eddy simulation of wind field
\[ \vec{u} = \vec{u} - \vec{u}' \]
\[ \frac{\partial \bar{n}_i}{\partial x_i} = 0 \]
\[ \tau_{ij} = \bar{u}_i \bar{u}_j - \bar{u}_i \bar{u}_j \]
\[ \frac{\partial \bar{u}_i}{\partial t} + \bar{u}_j \left( \frac{\partial \bar{u}_i}{\partial x_j} \right) = -\frac{1}{\rho} \frac{\partial \bar{p}}{\partial x_i} + \frac{1}{\rho} \bar{\Pi} + \mathbf{F}_i \]

Here, \( \vec{u} \) is the filtered (or resolved) velocity, \( u' \) is the residual (sub-grid scale) component, \( \tau_{ij} \) is the SGS stress tensor, \( F_i \) is a body force, and \( \Pi \) is the streamwise pressure gradient driving the flow:
\[ \frac{1}{\rho} \bar{\Pi} = -\frac{u_z^2}{L_z} \]

friction velocity

height of the computational domain

Wall-layer model for sea surface stress
\[ F_i = F_i + F'_i \]
\[ F_i = -\rho \int_A C_D \bar{u}_i R \left( \bar{u}_r \frac{\partial \bar{n}_r}{\partial x_j} \right) dxdy \]
\[ F'_i = -\rho \int_A \left[ \kappa \frac{\kappa}{\log(\delta_z / \sigma_w \sigma_{\bar{u}})} \right]^2 \| \bar{u}_r \| \| \bar{u}_r \| dxdy \]
\[ F_i = F_i + F'_i = -\rho \int_A \left( \kappa \ln^{-1} \frac{\delta_z}{\sigma_w \sigma_{\bar{u}}} \right)^2 \bar{u}_i R \left( \bar{u}_{r,j} \frac{\partial \bar{n}_r}{\partial x_j} \right) dxdy \]
\[ -\rho \int_A \left[ \kappa U_r(x,y,\delta_z) \right]^2 \bar{u}_{r,i}(x,y,\delta_z) U_r(x,y,\delta_z) dxdy \]

Scale invariant constraint
The scale invariant constraint requires the total drag force to be independent of the resolution. Thus, we dynamically obtain the drag by filtering \( F_i \) equation at two different filter scales: one is the LES filter scale of \( \Delta \) and the other is a test-filter scale of \( a\Delta \), where \( a > 1 \).

* This is the current state of our work, with the underlying theory finalized. Additional processing and analysis of the results are ongoing.